

Is the mass increase with velocity classical or relativistic phenomenon?

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Abstract

It has followed from experiments using cyclotrons that the inertial mass (or the resistance against velocity change) has been increasing with particle velocities. This increase has been interpreted as relativistic phenomenon of growing rest mass. However, there are no papers available where the relativistic predictions would be confronted with corresponding experimental data. It will be shown that the given increasing resistance against velocity change with increasing velocity may be interpreted also as a classical phenomenon. In such a case the velocity of matter objects would not be, however, limited by the light velocity; it might rise in principle to infinity. In this classical case it is not, of course, possible at the present to derive an exact dependence between energy and velocity. One may establish only a function dependence containing one free parameter (of velocity dimension) that is to be determined on the basis of corresponding experimental data. The given alternatives give, however, significantly different predictions. The comparison with experimental data should decide which of these two alternatives corresponds actually to the reality.

1. Introduction

The mass of individual objects may be interpreted according to the second law of Newton ($F = ma$) as the factor determining velocity change of an object when a force acts, while all objects remain in inertial motion when no forces exist [1]. However, in the theory of special relativity the corresponding factor has been correlated (through the value of light velocity) to the global energy of an object and has been interpreted as a quantity rising with the increasing energy of a given object. The given phenomenon has been discovered when in cyclotrons (or synchrocyclotrons or isochronous cyclotrons) the corresponding corrections had to be taken into account. The similar increase has played then important role in all other kinds of accelerators. However, it has not been ever presented that the relation between velocity and energy has corresponded really to relativistic predictions. Full satisfaction has consisted in that the value of light velocity has not been crossed in any case.

According to us some doubts should exist as a very strong increase should exist at relatively low energies (see the full line in Fig. 1 of this paper); the velocities being near to the light velocity should be reached for protons already at energies of approximately 10 GeV. And it has been started to speak about the velocities neighboring to light velocities only at TeV proton energies.

In the following we shall show that the given resistance increase against motion change may be described quite equivalently in the framework of classical physics when the quantity m in the second Newton law (characterizing the given resistance against motion change) is allowed to rise with velocity. And even if only a function dependence (containing one free parameter) between velocity and energy will be derived it is evident that the

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predictions in both the approaches (classical and relativistic) will be quite different. And the decision between them might be easily done on the basis of experimental data being already available in corresponding experimental groups.

2. Approach of Newton and classical physics

Let us start with the motion of a matter particle in a force field. It holds for a particle having velocity v :

$$dE_v = Fds, \quad vdv = a_v ds \quad (1)$$

where E_v is its energy and a_v - corresponding acceleration (change of velocity). According to Newton's law it must hold also

$$F = ma_v \quad (2)$$

where m is the value characterizing corresponding properties of a given particle (i.e., the ability of a force to change particle velocity). It follows then from the preceding equations

$$dE_v = mv dv. \quad (3)$$

Integrating the last equation one may write (if m is constant and acceleration is independent of velocity)

$$E_v = E_0 + \frac{1}{2}mv^2$$

where the integration constant E_0 may be interpreted as a basic energy value corresponding to velocity $v = 0$. And one should ask how to interpret this constant. There is not any reason to regard the original choice of classical physics (i.e., $E_0 = 0$) as the only possibility. The increase of kinetic energy is given by the acting force but the global energy value of an object (relating to force effect) need not be determined by the instant value of velocity only.

It is evident that solutions with non-zero E_0 should be equally acceptable. In such a case it is suitable to write $E_0 = mc^2$ where c is a free constant parameter of velocity dimension. It enables to correlate the given integration constant to the resistance against velocity change, which will be made use of in the following. The constant c has nothing to do with light velocity; and also the interpretation of parameter m (as to the corresponding mass) should be newly considered. If $c > 0$ it is possible to write

$$E_v = m(c^2 + \frac{1}{2}v^2) = m_v c^2. \quad (4)$$

It is then possible to write further

$$m_v = m(1 + \frac{1}{2}\beta^2) \quad (5)$$

where $\beta = v/c$; it holds also

$$m = m_0$$

where m_0 may be interpreted now mainly as the physical characteristic representing the resistance against the motion change at zero velocity.

However, it has been known from accelerator experiments that the resistance m against motion at higher velocities is not constant (as already mentioned). Thus, the quantity m in Eq. (2) is to be substituted by a new quantity (i.e., by m_v) rising with velocity v . The

corresponding general problem will be analyzed in the next section.

3. Generalized approach in classical physics²

We shall now assume that instead of Eq. (2) it will hold

$$F = m_v a_v, \quad (6)$$

i.e., both the factors on the right-hand side are taken as v -dependent. However, in such a case it is necessary to write instead of Eq. (5)

$$m_v = m_0 f(\beta) \quad (7)$$

where $f(\beta)$ is a function rising with β ; $f(0) = 1$. And if it is required in the analogy to Eq. (4) to hold again

$$E_v = m_v c^2 \quad (8)$$

one can write (see Eqs. (3) and (6))

$$m_v v dv = dE_v = c^2 dm_v,$$

which represents the condition for determining function $f(\beta)$ if c is taken as a free constant that should be determined from experimental data. Using Eq. (7) and introducing $v = \beta c$ one obtains

$$\beta f(\beta) d\beta = df(\beta)$$

or

$$f(\beta) = e^{\frac{1}{2}\beta^2}, \quad (9)$$

which means that Eq. (7) may be interpreted as the generalization of Eq. (5); holding for all values of v . The resistance against the velocity change increases now exponentially with v^2 . The expression given in Eq. (5) represents then the first approximation for very small values of v . Parameter c may be a non-zero positive constant bringing E_0 to a non-zero value, at the difference to the earlier classical (Newton) case where E_0 has been put to equal zero, and E_v has been taken to represent kinetic energy only.

We may denote Eq. (7) (with Eq. (9)) as the generalized classical solution, differing substantially from the formula obtained in the relativistic alternative. And we should ask which value of c might correspond to experimental facts as there is not any reason now for being equal to light velocity, at the difference to the theory of special relativity [2].

4. Relativistic approach

It follows from the preceding that Eqs. (8) and (7) have defined the function $f(\beta)$ that determines the relation between velocity v and total energy E_v of a given particle. And we have derived the dependence (given by Eq. (9)) that should hold in the generalized classical approach. In such a case the ratio between the force F and acceleration a_v being imparted to a particle by this force at velocity v is equal to its inertial mass m_v .

However, this ratio may be represented quite generally by M_v , i.e. one may write

$$F = M_v a_v,$$

²The basic idea has been published already earlier (/arXiv:0711.0061[physics.gen-ph]); however, the whole approach has not been sufficiently reasoned.

where M_v is represented by an arbitrary function depending on v (or on β). In the general classical approach it holds $M_v = m_v$, which does not hold of course, e.g., in the case of special relativity. In this case instead of Eq. (9) the following function $f^{(r)}(\beta)$ is being made use of:

$$f^{(r)}(\beta) = \frac{1}{\sqrt{1 - \beta^2}} \quad (10)$$

where it is assumed that it holds $v/\beta = c_l$, i.e., the earlier parameter c being equal to light velocity. Using Eqs. (1) and also Eqs. (6-8) (where m_v is substituted by M_v) one can write

$$M_v = \frac{1}{v} \frac{dE_v}{dv} = \frac{M_0}{\beta} \frac{df^{(r)}(\beta)}{d\beta}.$$

Parameter M_v characterizes the relation between the acting force and acceleration. However, instead of m_v (see Eqs. (7) and (9)) one obtains

$$M_v = \frac{M_0}{(1 - (\frac{v}{c})^2)^{3/2}}. \quad (11)$$

where $M_0 = m_0$. Introducing the expression for relativistic inertial mass

$$m_v^{(r)} = \frac{m_0}{\sqrt{1 - \beta^2}} \quad (12)$$

it is possible to write

$$M_v = \frac{d}{dv}(m_v^{(r)}v). \quad (13)$$

And it holds for the effect of the force

$$F = M_v \frac{dv}{dt} = \frac{d}{dt}(m_v^{(r)}v); \quad (14)$$

or: the force size corresponds to the time change of particle momentum, which differs from Eq. (6); and also the first relation in Eq. (1) is not fulfilled. Kinematic force effects to motion characteristics of matter objects are in classical and relativistic alternatives significantly different.

5. Velocity increase with energy

There are, therefore, two different (classical and relativistic) formulas (9) and (10) (see also Eq. (7)) characterizing the effect of the force in the dependence on the velocity of moving object. For the dependence of the velocity on energy one can write then in the classical case

$$v = c \sqrt{2 \ln(\frac{E_v}{E_0})} \quad (15)$$

and in the relativistic case

$$v = c_l \sqrt{1 - (\frac{E_0}{E_v})^2}; \quad (16)$$

where c is a free constant parameter (to be determined from experimental data) and c_l corresponds to the light velocity.

The corresponding velocity dependencies on rising energies (for accelerated protons) are represented in Fig. 1. The ratio E_v/E_0 is given on the horizontal axis and the

velocity v is shown on the vertical axis; being given in light velocity units. The full line represents the relativistic case. The other three dependencies (represented by dashed lines) correspond then to the classical alternative: $c = c_l$ (upper line), $c_l/3$ (middle line) and $c_l/6$ (lower line).

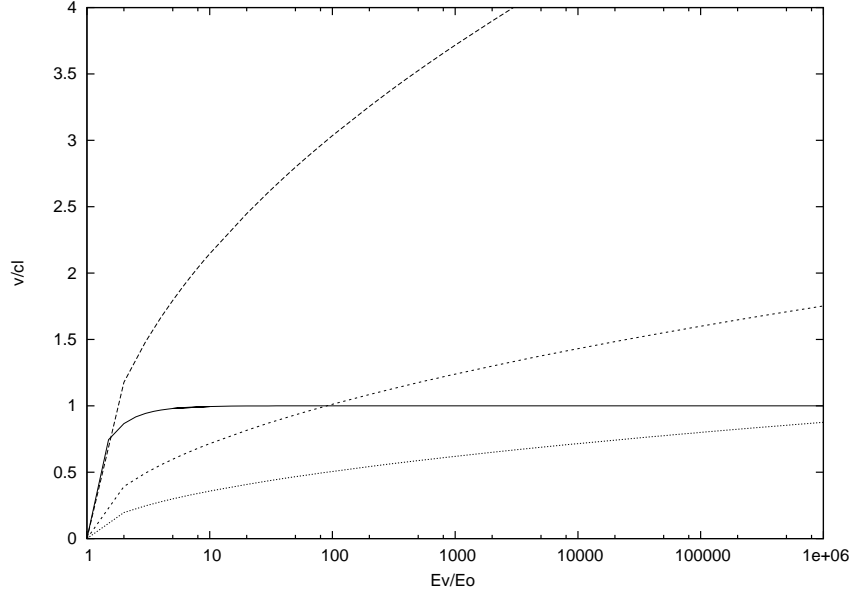


Figure 1: *The velocity increase in the dependence on the energy of moving object; the ratio E_v/E_0 is given on the horizontal axis, the velocity in units of light velocity, i.e. v/c_l , on vertical axis. Full line - behavior according to relativity theory, dashed lines - three different classical possibilities (characterized by different c values) as the actual value of c should be established from experimental data (see text).*

In the relativistic case the value of parameter c_l is uniquely defined and the velocity v should go quickly to the light velocity. E.g., in the case of protons the values near to the light velocity should be obtained already at values less than 10 GeV and for electrons at 5 MeV (see Fig.1 - full line). And it is very improbable that this fact may be brought to harmony with data obtained in the case of cyclotrons where the rise of inertial mass (resistance against motion change) was discovered for the first time.

In the classical case the constant c is a free parameter that should be derived from experimental data that have been gained in principle already with the help of many different accelerators. As the velocity of light has not been crossed until now the velocity of matter objects should rise with increasing energy E_v more slowly than in the relativistic case but permanently.

Some very approximative estimation of parameter c value may be done with the help of the results published in the very recent days. It has been announced by the group of OPERA neutrino experiment [3] that the light velocity has been reached at neutrino energy of 17 GeV. Making use the more previous estimation [4] of the neutrino rest energy, i.e., $E_0 = 2\text{eV}$, one might obtain from Eq. (15) $c \approx c_l/6$. However, if one assumed that the light velocity has been practically obtained for protons at energy of 3.5 GeV (as a higher value has not been announced until now) one should obtain practically the highest possible value $c \approx c_l/4$. In such a case the neutrino rest mass value would correspond

approximately to 5 eV and the electron would reach light velocity at cca 1.75 GeV. And the velocity of protons at, e.g., 550 GeV, resp. 7 TeV, would be approximately 87%, resp. 104%, of the light velocity. However, more precise value of parameter c may be determined when it will be derived from real values of particle velocities reached in individual accelerators.

6. Conclusion

The increase of particle mass (considered as relativistic phenomenon until now) may be interpreted as the increase of the resistance against velocity change with increasing velocity also in the framework of classical physics. However, the velocity dependence differ fundamentally from that derived in the framework of special relativity theory. The decision between these two alternatives may be done on the basis of experimental data that are already available in different experimental groups.

However, at this place it might be interesting to mention that a similar value of velocity parameter (being little less than light velocity and in principle of the same order) was obtained by Maxwell when he joined electrical and magnetic characteristics into one common system. The identification of the given parameter with light velocity was done rather arbitrarily (from the neighborhood of these two values) without any deeper reason (see [5]). It may be seen that the velocity parameter of Maxwell and that of ours represent in principle the same physical reality, i.e., the relation between static (rest) and dynamical characteristics of corresponding objects.

Even if the decision between different possible values of c must be given on the experimental basis the increasing resistance against velocity change with rising velocity may be well understood in the framework of classical ontological interpretation (Galileo, Newton) on the basis of inertia principle. However, some new questions might be opened: What is the actual reason of this changing resistance? And further: How is the force transmitted to an object in "physical vacuum"? However, first the decision between different alternatives on experimental grounds should be done.

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